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INVENTORY MODEL FOR DECAYING ITEMS WITH STOCK DEPENDENT DEMAND OF MONEY UNDER PERMISSIBLE DELAY IN PAYMENTS

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ABSTRACT

Inventory control has one of the most important tasks faced by modern manager. The investment in inventories for most form their assets committed to inventories. Further inventories one often the least stable and difficult to manage type of assist. Rapid change in level of business activities effect on inventories. In recent year, change in interest rate effect the inventories. Employ and customer theft has also led to increased cost of maintaining inventories. But carrying inventory is a costly thing as the storage cost, stock out cost, capacity related cost, item cost, ordering cost, deterioration and expiration of the product etc. must be taken in to account. Some policies, procedures and techniques employed in maintaining the optimum number of amount of each inventory item is the inventory management. While inventory is an asset, it is a non productive asset since it earns no interest but costs an organization in handling insurance, taxes, shrinkage and space. Careful inventory management can make a huge difference in the profitability of a firm.

KEY WORDS*:* Inventory, modern manager, policies

INTRODUCTION

Classical deterministic inventory models consider the demand rate to be either constant or time-dependent but independent of the stock status. However, for certain types of consumer goods (e.g., fruits, vegetables, donuts and other) of inventory, the demand rate may be influenced by the stock level. It has been noted by marketing researchers and practitioners that an increase in a product's shelf space usually has a positive impact on the sales of that product and it is usually observed that a large pile of goods on shelf in a supermarket will lead the customer to buy more, this occurs because of its visibility, popularity or variety and then generate higher demand. In such a case, the demand rate is no longer a constant, but it depends on the stock level. This phenomenon is termed as 'stock dependent consumption rate'. In general, 'stock dependent consumption rate' consists of two kinds. One is that the consumption rate is a function of order quantity (initial stock level) and the other is that the consumption rate is a function of inventory level at any instant of time.

The consumption rate may go up or down with the on-hand stock level. These phenomena attract many marketing researchers to investigate inventory models related to stock-level. Conversely, low stocks of certain baked goods (e.g., donuts) might raise the perception that they are not fresh. Therefore, demand is often time and inventory-level dependent.

This paper strives; an inventory model for deteriorating items with multi variate demand rate under inflationary environment. We have taken a more realistic demand rate that depends on two factors, one is time, and the second is the stock level available. The stock level in itself obviously gets depleted due to the customer's demand. As a result, what we witness here is a circle in which the customer's demand is being influenced by the level of stocks available, while the stock levels are getting depleted due to the customer's demands. This assumption takes the customer's interests as well as the market forces into account. The demand rate is such that as the inventory level increases, it helps to increase the demand for the inventory under consideration. While as the time passes, demand is depends upon the various factors. The competitive nature of the market has been accounted for by taking permissible delay in payments into consideration. Finally, the results have been illustrated with the help of numerical examples. Also, the effects of changes of different parameters are studied graphically.

ASSUMPTIONS AND NOTATIONS:

The mathematical models of the two warehouse inventory problems are based on the following assumptions and notations:

Assumptions:

- The inventory system involves a single type of items.
- Demand rate is dependent on time and stock level.
- Deterioration rate is taken as Kt.
- Shortages are not permitted.
- The replenishment rate is instantaneous.
- Lead time is neglected.
- Permissible delay in payment to the supplier by the retailer is considered. The supplier offers different discount rates of price at different delay periods.
- Planning horizon is infinite.
- Inflation and time value of money is considered.

Notations:

- $D = a + bt + cI(t)$ Time and Stock dependent demand
- C_O = Ordering cost
- C_h = holding cost per unit time, excluding interest charges
- C_P = purchasing cost which depends on the delay period and supplier's offers
- p = selling price per unit
- M = permissible delay period
- M_i = i^{th} permissible delay period in settling the amount
- $_i$ = discount rate (in %) of purchasing cost at i-th permissible delay period.
- i_e = rate of interest which can be gained due to credit balance
- i_c = rate of interest charged for financing inventory
- T = length of replenishment
- $AP_1(T, M_i)$ = average profit of the system for $T \geq M_i$
- $AP_2(T, M_i)$ = average profit of the system for $T \leq M_i$
- Q_0 = Initial lot size

MODEL FORMULATION AND SOLUTION

The cycle starts with initial lot size Q_0 and ends with zero inventory at time t=T. Then the differential equation governing the transition of the system is given by

$$\frac{dI(t)}{dt} = -Kt - (a + bt + cI(t)), \qquad 0 \leq t \leq T \qquad \text{...} \qquad (1.1)$$

With boundary condition $I(0) = Q_0$

The purchasing cost at different delay periods are

$$C_P = \begin{cases} C_r(1-\delta_1), M = M_1 \\ C_r(1-\delta_2), M = M_2 \\ C_r(1-\delta_3), M = M_3 \\ \infty, M > M_3 \end{cases}$$

Where C_r = maximum retail price per unit.

And M_i (i=1,2,3) = decision point in settling the account to the supplier at which supplier offers δ % discount to the retailer.

Now two cases may occur:

1. When $T \geq M$
2. When $T < M$

Case 1: when $T \geq M$

Solving the equation (1), we get

$$\frac{dI(t)}{dt} + KtI(t) = -(a + bt + cI(t))$$

Using the boundary condition $I(0) = Q_0$, we get

$c = Q_0$

Therefore the solution of equation (1) is

$$I(t) = \left\{ Q_0 - at - \frac{(a+b)}{2}t^2 - \left(\frac{aK}{2} + b\right)\frac{t^3}{3} - \frac{bK}{8}t^4 \right\} e^{-t-Kt^2/2} \qquad 0 \leq t \leq T \qquad \text{... (1.2)}$$

In this case it is assumed that that the replenishment cycle T is larger than the credit period M.

The holding cost, excluding interest charges is

$$HC = C_h \int_0^T I(t) e^{-rt} dt$$

$$\begin{aligned} HC = C_h \Bigg[& \left\{ Q_0 T - \frac{a}{2}T^2 - \frac{(a+b)}{6}T^3 - \left(\frac{aK}{2} + b\right)\frac{T^4}{12} - \frac{bK}{40}T^5 \right\} \\ & - (1+r)\left\{ \frac{Q_0}{2}T^2 - \frac{a}{3}T^3 - \frac{(a+b)}{8}T^4 - \left(\frac{aK}{2} + b\right)\frac{T^5}{15} - \frac{bK}{48}T^6 \right\} \\ & - \frac{K}{2}\left\{ \frac{Q_0}{3}T^3 - \frac{a}{4}T^4 - \frac{(a+b)}{10}T^5 - \left(\frac{aK}{2} + b\right)\frac{T^6}{18} - \frac{bK}{56}T^7 \right\} \Bigg] \end{aligned} \qquad \text{... (1.3)}$$

The cost of financing inventory during time span [M,T] is

$$FC = i_c C_P \int_M^T I(t) e^{-r(M+t)} dt$$

$$\begin{aligned} FC = i_c C_P \Bigg[& (1 - rM)\left\{ Q_0 T - \frac{a}{2}T^2 - \frac{(a+b)}{6}T^3 - \left(\frac{aK}{2} + b\right)\frac{T^4}{12} - \frac{bK}{40}T^5 \right\} \\ & - (1+r)\left\{ \frac{Q_0}{2}T^2 - \frac{a}{3}T^3 - \frac{(a+b)}{8}T^4 - \left(\frac{aK}{2} + b\right)\frac{T^5}{15} - \frac{bK}{48}T^6 \right\} \\ & - \frac{K}{2}\left\{ \frac{Q_0}{3}T^3 - \frac{a}{4}T^4 - \frac{(a+b)}{10}T^5 - \left(\frac{aK}{2} + b\right)\frac{T^6}{18} - \frac{bK}{56}T^7 \right\} \end{aligned}$$

$$-(1-rM)\left\{Q_0M-\frac{a}{2}M^2-\frac{(a+b)}{6}M^3-\left(\frac{aK}{2}+b\right)\frac{M^4}{12}-\frac{bK}{40}M^5\right\}.$$
$$+(1+r)\left\{\frac{Q_0}{2}M^2-\frac{a}{3}M^3-\frac{(a+b)}{8}M^4-\left(\frac{aK}{2}+b\right)\frac{M^5}{15}-\frac{bK}{48}M^6\right\}$$
$$+\frac{K}{2}\left\{\frac{Q_0}{3}M^3-\frac{a}{4}M^4-\frac{(a+b)}{10}M^5-\left(\frac{aK}{2}+b\right)\frac{M^6}{18}-\frac{bK}{56}M^7\right\}\Bigg] \quad \text{... (1.4)}$$

Opportunity gain due to credit balance during time span [0,M] is

$$Opp.Gain = i_e p\int_0^M (M-t)(a+bt+cI(t))e^{-rt}dt$$
$$Opp.Cost = i_e p\left[(a+bM)\frac{e^{-rM}}{r^2}-2b\frac{e^{-rM}}{r^3}+\frac{aM}{r}-\frac{(a-bM)}{r^2}+\frac{2b}{r^3}\right] \quad \text{... (1.5)}$$

Therefore, the total cost is given by

TC_{1i}=Purchasing Cost +holding cost +ordering cost +interest charged-interest earned for M$\epsilon\{M_1,M_2,M_3\}$

$$TAC_{1i}=\frac{1}{T}TC_{1i} \quad \text{... (1.6)}$$

Case 2 when T < M

In this case, credit period is larger than the replenishment cycle consequently cost of financing inventory is zero. The holding cost, excluding interest charges is

$$HC = C_h\int_0^T I(t)e^{-rt}dt$$
$$HC = C_h\left[\left\{Q_0T-\left(\frac{a}{2}T^2+\frac{b}{6}T^3+\frac{aK}{24}T^4+\frac{bK}{40}T^5\right)\right\}\right.$$
$$-r\left\{\frac{Q_0}{2}T^2-\left(\frac{a}{3}T^3+\frac{b}{8}T^4+\frac{aK}{30}T^5+\frac{bK}{48}T^6\right)\right\}$$
$$\left.-\frac{K}{2}\left\{\frac{Q_0}{3}T^3-\left(\frac{a}{4}T^4+\frac{b}{10}T^5+\frac{aK}{36}T^6+\frac{bK}{56}T^7\right)\right\}\right] \quad \text{...(1.7)}$$

Opportunity gain due to credit balance during time span [0,M] is

$$Opp.Gain = i_e p\left[\int_0^T (T-t)(a+bt)e^{-rt}dt+\int_0^T (M-T)(a+bt)e^{-rt}dt\right]$$
$$= i_e p\left[\int_0^T \{aT+(bT-a)t-bt^2\}e^{-rt}dt+(M-T)\int_0^T\{a+bt\}e^{-rt}dt\right] \quad \text{..... (1.8)}$$

Therefore the total cost during the time interval T is given by

TC_{2i}=Purchasing cost +holding cost +ordering cost-interest earned (Opp. cost)

$$TAC_{2i}=\frac{1}{T}TC_{2i} \quad \text{..... (1.9)}$$

Now, our aim is to determine the optimal value of T and M such that TAC(T,M) is minimized where

$$TAC(T,M) = Inf.\begin{cases} TAC_{1i}(T,M), TAC_{2i}(T,M) \\ where, M\varepsilon(M_1, M_2, M_3) \end{cases} \qquad ..... (1.10)$$

Special case:

Case 1 when there is no deterioration, i.e. K=0, then

$$I(t) = \left\{Q_0 - \left(at + \frac{b}{2}t^2\right)\right\}, \qquad 0 \le t \le T$$

$$\mathrm{FC} = \mathrm{i_c C_P}\left[\left\{Q_0 T - \left(\frac{a}{2}T^2 + \frac{b}{6}T^3\right)\right\} - rM\left\{Q_0 - \left(aT + \frac{b}{2}T^2\right)\right\}\right.$$
$$- \frac{K}{2}\left\{\frac{Q_0}{3}T^3 - \left(\frac{a}{4}T^4 + \frac{b}{10}T^5\right)\right\} - r\left\{\frac{Q_0}{2}T^2 - \left(\frac{a}{3}T^3 + \frac{b}{8}T^4\right)\right\}$$
$$- \left\{Q_0 M - \left(\frac{a}{2}M^2 + \frac{b}{6}M^3\right)\right\} + rM\left\{Q_0 - \left(aM + \frac{b}{2}M^2\right)\right\}$$
$$\left. + \frac{K}{2}\left\{\frac{Q_0}{3}M^3 - \left(\frac{a}{4}M^4 + \frac{b}{10}M^5\right)\right\} + r\left\{\frac{Q_0}{2}M^2 - \left(\frac{a}{3}M^3 + \frac{b}{8}M^4\right)\right\}\right]$$

$$Opp.Cost = i_e p\left[(a + bM)\frac{e^{-rM}}{r^2} - 2b\frac{e^{-rM}}{r^3} + \frac{aM}{r} - \frac{(a - bM)}{r^2} + \frac{2b}{r^3}\right]$$

Case 2: when the demand rate is constant means b=0

$$I(t) = \left\{Q_0 - \left(at + \frac{aK}{6}t^3\right)\right\}e^{-Kt^2/2} \qquad 0 \le t \le T$$

$$\mathrm{HC} = \mathrm{C_h}\left[\left\{Q_0 T - \left(\frac{a}{2}T^2 + \frac{aK}{24}T^4\right)\right\} - r\left\{\frac{Q_0}{2}T^2 - \left(\frac{a}{3}T^3 + \frac{aK}{30}T^5\right)\right\}\right.$$
$$\left. - \frac{K}{2}\left\{\frac{Q_0}{3}T^3 - \left(\frac{a}{4}T^4 + \frac{aK}{36}T^6\right)\right\}\right]$$

$$Opp.Cost = i_e p\left[a\frac{e^{-rM}}{r^2} + \frac{aM}{r} - \frac{a}{r^2}\right]$$

Table 1: Variation in TC with the variation in a

a	T	TC(10^5)
70	31.7643	6.5247
80	34.1886	6.1436
90	33.9981	1.7245
100	32.0002	1.2681
110	32.8266	1.8957
120	30.1724	1.3541
130	21.9610	4.5232

Table 2: Variation in TC with the variation in b

b	T	TAC(10^5)
30	21.235	10.2954
35	28.1254	10.1118
40	30.4457	9.9725
45	33.1896	8.1829
50	33.7832	7.2681
55	34.1457	7.0075
60	34.8485	6.3221
65	31.9517	6.1882
70	36.1725	6.0914
75	38.8954	1.3236

Table 3: Variation in TC with the variation in C_h

C_h	T	TAC(10^5)
0.020	18.7241	1.8954
0.025	21.9154	2.3112
0.030	21.1892	2.7776
0.035	27.2431	3.8154
0.040	29.8561	4.1112
0.045	31.1452	4.5772
0.050	32.0002	1.2681
0.055	33.5231	6.3314
0.060	34.1272	6.7821
0.065	31.4139	7.1957

Table 4: Variation in TC with the variation in K

K	T	TAC(10^5)
0.0008	32.8081	4.5417
0.0009	32.8081	4.9857
0.0010	32.8081	1.2681
0.0015	32.8081	6.8934
0.0020	32.8081	6.9572
0.0025	32.8081	7.2231
0.0030	32.8081	7.8573
0.0035	32.8081	9.4315
0.0040	32.8081	10.5473
0.0045	32.8081	11.1892

CONCLUSION

In this paper we developed a model with supplier's trade offer of credit and price discount the purchase of stock. The model considered the both, deterioration effect and time discounting. Generally, supplier offer different price discount on purchase of items of retailer at different delay periods. Suppliers allow maximum delay period, after which they will not take a risk of getting back money from retailers or any other loss of profit. Constant deterioration is not a viable concept; hence, we have considered an inventory with deterioration increasing with time. To make our study more suitable to present-day market, we have done our research in an inflationary environment. In totality, the fact that the whole study has been done under the implications of inflation, gives it a viability that makes it more pragmatic and acceptable. The setup that has been chosen boasts of uniqueness in terms of the conditions under which the model has been developed.

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REFERENCES

1. Pal, A.K. (2002). "A note on an inventory model with inventory level dependent demand rate", *Journal of the Operational Research Society; 41:971–971.*
2. Baker, R.C., Urban, T.L. (1988). "A deterministic inventory system with an inventory level-dependent demand rate", *Journal of the Operational Research Society; 39 (9): 823–831.*
3. Bar-Lev, S.K., Parlar, M., Perry, D. (1994). "On the EOQ model with inventory-level-dependent demand rate and random yield", *Operations Research Letters; 16: 167–176.*
4. Bhunia, A. K., Maiti, M. (1994). "A two warehouses inventory model for a linear trend in demand", *Opsearch; 31: 318–329.*
5. Goswami, A., Chaudhuri, K. S. (1992). "An economic order quantity model for items with two levels of storage for a linear trend in demand", *Journal of the Operational Research Society; 43:157–167.*
6. Hsieh, T.P., Dye, C.Y. and Ouyang, L.Y. (2007). "Determining optimal lot size for a two- warehouse system with deterioration and shortages using net present value", *European Journal of Operational Research; 191 (1): 180-190.*
7. Sharma, A.K. (2010). "On deterministic production inventory model for deteriorating items with an exponential declining demand", *Acta Ciencia Indica, XXVI, 4, 305-310.*
8. Yang, H.L., (2004): "A two warehouse inventory model for deteriorating items with shortages under inflation", *European Journal of Operational Research; 157: 344-356.*
9. Bansal K.K.[2013] Inventory Model For Deteriorating Items With The Effect of Inflation. *International Journal of Application and Innovation in Engineering and Management.2[5] 143-150*
10. Bansal K.K., Ahlawat N. [2012] Integrated Inventory Models for Decaying Items with Exponential Demand under Inflation. *International Journal of Soft Computing and Engineering (IJSCE) 2[3] 578-587*
11. Bharat G., Bansal K.K. [2015] A Deterministic of Some Inventory System For Deteriorating Items With An Inventory Level Dependent Demand Rate. *International Journal of Education and Science Research Review2[6] 94-96*
12. Bansal K.K. [2016] A Supply Chain Model With Shortages Under Inflationary Environment. *Uncertain Supply Chain Managemen*t 4[4] 331-340
13.
14. Uthayakumar, R.(2013). "An EOQ model for perishable items under stock and time-dependent selling rate with shortages", *ARPN Journal of Engineering and Applied Sciences, 4, 3:8-14.*
15. Levin, R.I., McLaughlin, C.P., Lamone, R.P., Kottas, J.F. (1972). "Production/ Operations Management: Contemporary Policy for Managing Operating Systems", *McGraw-Hill, New York.*
16. Phaujdar, S. (2004). "An inventory model for deteriorating items and stock-dependent consumption rate", *Journal of the Operational Research Society; 40: 483–488.*
17. A K Malik , Dipak Chakraborty , Kapil Kumar Bansal, Satish Kumar (2017) : "Inventory Model with Quadratic Demand under the Two Warehouse Management System" International Journal of Engineering and Technology (IJET) Vol.3 (3) pp.2299-2303
18. Bansal K.K ,Pravinder Kumar (2016) : POSITION OF SUPPLY CHAIN MANAGEMENT IN INDIAN INDUSTRY, *International Journal of Education and Science Research ReviewVol.3 (3)*
19. M., Maiti. (2014). "Two storage inventory model with random planning horizon", *Applied Mathematics and Computation; 183(2): 1084-1097.*
20. Ray, J. and Chaudhuri, K.S. (1997). "An EOQ model with stock-dependent demand, shortage, inflation and time discounting", *International Journal of Production Economics; 53(2): 171-180.*
21. Peterson, R. (2005): "Decision systems for inventory management and production planning, 2nd ed", *Wiley, New York.*